# SHEAR CRACK PROPAGATION IN THE RESIDUAL STRESS FIELD

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The problem of propagation of a longitudinal shear crack in a medium with a random field of internal stresses is considered and solved with the use of the theory of quasi-brittle failure. Local criterion of crack propagation under cyclic loading is derived, and its application as a model of fatigue crack propagation is investigated.

Conditions of longitudinal shear imply that displacements are normal to a certain xy-plane in the absence of external loads. In an elastic medium such a field can only exist if the resulting strains do not satisfy the compatibility condition.

It can be shown that any internal stress field can be attributed to the distribution of dislocations [1]. Under conditions of longitudinal shear a strain field which does not satisfy the compatibility condition can be defined in terms of helical dislocations whose axes are normal to the xy-plane. When a crack or cavity is formed, the initial field of internal stresses is converted, owing to changed boundary conditions, into a certain stress field which we shall call residual. During the process of crack or cavity propagation the dislocation distribution density will be considered to be an invariant characteristic.

Let us consider in the plane of the complex variable $z = x + iy$ a semiinfinite crack resulting from longitudinal shear (Fig. 1) and determine the stress intensity coefficient k at the crack tip $z = -h$.

Using the conformal transformation $z = \omega(\zeta)$ of the region around the crack into the half-plane $\eta > 0$ of variables $\zeta = \xi = i\eta$, we obtain

$$k = i\frac{GF'(\zeta)}{\sqrt{\omega''(\zeta)}} \qquad \text{for} \quad \zeta = 0 \tag{1}$$

Here $F(\zeta)$ is a function of stress in the mapped region, $\zeta = 0$ is the point which in the conformal map corresponds to the crack tip, and the prime denotes differentiation.

For an individual helix of dislocation at point $z_0 = \alpha + i\beta$ the stress function is of the form

$$F(\zeta) = ib \ln \frac{\zeta - \zeta_0}{\zeta - \bar{\zeta}_0} \tag{2}$$

where b is the Burgers vector and $\zeta_0 = \xi_0 + i\eta_0$ is the point defined by the transformation $z_0 = \omega(\zeta_0)$. Function $z = \omega(\zeta) = -h-\zeta^2$ is used for conformal mapping the region surrounding the crack in the halfplane in which the crack tip is represented by point $\zeta = 0$.

From Eqs. (1) and (2) follows

$$k = bG\sqrt{2}\,\eta_0 / (\xi_0^2 + \eta_0^2) \tag{3}$$

Using equation $z_0 = -h-\zeta_0^2$, we obtain

$$k = bG\left(\frac{\alpha + h + \sqrt{(\alpha + h)^2 + \beta^2}}{(\alpha + h)^2 + \beta^2}\right)^{1/2} \tag{4}$$

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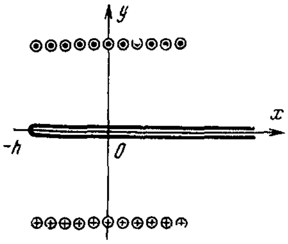


Fig. 1

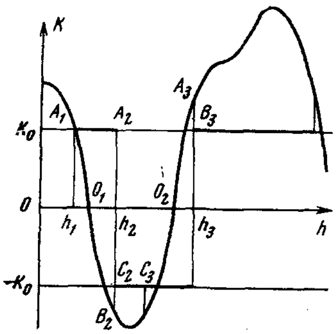


Fig. 2

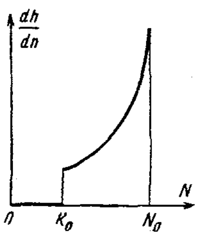


Fig. 3

If the field of distributed dislocations is defined by the density of Burgers vectors $\rho(\alpha, \beta)$, the coefficient of stress intensity is calculated by formula

$$k = G \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(\alpha, \beta) \left( \frac{\alpha + h + \sqrt{(\alpha + h)^2 + \beta^2}}{(\alpha + h)^2 + \beta^2} \right)^{1/2} d\alpha \, d\beta \tag{5}$$

Let us assume that the field $\rho(\alpha, \beta)$ is stochastic and statistically homogeneous [3]. The position of the crack (tip) is defined by parameter h and the relationship between stochastic functions $\rho(\alpha, \beta)$ and k(h) by expression (5). Passing to variables $t = \alpha + h$, we obtain

$$k(h) = G \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(t - h, \beta) \left( \frac{t + \sqrt{t^2 + \beta^2}}{t^2 + \beta^2} \right)^{1/2} dt \, d\beta \tag{6}$$

The parallel translation $t = \alpha + h$ does not alter the probability characteristics of function $\rho(\alpha, \beta)$. Since the integral transformation (5) of function $\rho(t-h, \beta)$ is independent of h, hence h(k) is a statistically stationary function.

Let us consider the crack propagation process under combined residual stresses and external loads. As the condition of crack propagation we take the criterion of Griffith-Irwin, which in this case can be written as

$$|k(h) + R| = k_0 \tag{7}$$

where $k_0$ is the value of the intensity coefficient and R is the coefficient of intensity of stresses produced by external loads.

Let the intensity coefficient R be produced by alternating external loads and its maximum and minimum absolute values equal. Let us consider the crack propagation process on the assumption that the extreme value of R is equal to N.

One of the patterns of the stochastic function k(h) is shown in Fig. 2. Its value for h = 0 corresponds to the initial position of the crack (tip). For R = 0 the initial position of the crack (tip) is unstable, since $k > k_0$, and the crack propagates until it becomes stabilized (with the tip) at the $h = h_1$. Further growth of the crack depends on changes of external loads.

When with $N < k_0$ the crack tip reaches points $O_1$ or $O_2$, then any variation of external loads within the limits $-N \leq R \leq N$ will always result in the stabilization of the crack.

If $N = k_0$, the crack (tip) can be brought to point $O_1$ by increasing R to $k_0$. The position of the crack (tip) will then be stable, since a further development of the crack under fixed external loads would result in a decrease of the overall coefficient of intensity. Under decreasing external loads the crack tip will remain at point $O_1$; however, on reaching $R = -k_0$, the crack becomes unstable, since its further growth would lead to an increase of the absolute value of the overall intensity coefficient. With the crack (tip) at point $O_2$ the crack becomes stable again.

This process is repeated at each single cycle of load variation and the crack is extended by twice the distance between the zeros of function k(h). The rate of crack growth depends on the number of cycles n and can be expressed by

$$\frac{dh}{dn} = \frac{2}{Q} \qquad (8)$$

where Q is the number of zeros of function k(h) per unit of length.

In a certain sense the dependence (8) of crack growth on the number of cycles may be called unstable, since the smallest deviation toward $N < k_0$ results in the stabilization of the crack. A finite rate of crack propagation obtains when the equality $N = k_0$ is strictly satisfied.

Let us consider the case of $N > k_0$, and assume that from the instant of incipient crack development at $h = h_1$, R increases to N. This results in an increase of the crack with its tip reaching $h = h_2$. In Fig. 2 the length of segment $A_2B_2$ corresponds to N.

Further reduction of external load results in the unstable state $C_2$ which changes to state $C_3$. When the load is decreased to the extent that $R = -N$, the crack (tip) reaches point $h_3$. The subsequent increase of R leads to the unstable state $B_3$, and so on. Points $B_2$, $A_2$, etc. correspond to sudden increases of the crack equal to the distance between the (points of) intersection of function k(h) with the level $|N-k_0|$.

Let $P(N-k_0)$ be the average number of overshoots of the stochastic function k(h) beyond the level $|N-k_0|$, then the rate of crack propagation can be written in the form

$$\frac{dh}{dn} = \frac{2}{P(N-k_0)} \qquad (9)$$

When $P(N-k_0) = 0$, residual stresses cannot restrain the development of the crack, and the rate dh/dn becomes infinite.

Let $N_0$ correspond to the case in which the number of intersections between k(h) and the $|N-k_0|$ level is zero. Then, when the coefficient of stress intensity reaches $N_0$, the cyclic growth of the crack develops into a brittle fracture.

If one assumes that the number of overshoots of function k(h) monotonically decreases with the increase of the $|N-k_0|$ level, the rate of crack development can be qualitatively represented in terms of N by the curve shown in Fig. 3.

It follows from Eqs. (4) and (5) that the effect of internal stresses on the intensity coefficient k is inversely proportional to the distance to the crack tip. If the dimensions of the crack are considerable in comparison with the correlation radius of the field of internal stresses, Eq. (9) can be considered as the local criterion of crack propagation under cyclic loading.

The dependence of dh/dn on N shown in Fig. 3 can be used for defining the growth of fatigue cracks. In the range of N – limited by experimental conditions – a power dependence is usually assumed [4, 5]. However at fairly small values of N the crack growth can become unstable, and may altogether cease. In Fig. 3 this corresponds to $N \leq k_0$.

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